

Appendix: Effective Tax Rates with Pillar Two Top-up Minimum Taxes

Qualified Tax Incentives by Instrument Type*

Shafik Hebous[†] and Andualem Mengistu[‡]

January 2026

Abstract. This appendix presents the derivations of forward-looking effective tax rates under the Pillar Two minimum tax in the presence of qualified tax incentives, as implemented in the accompanying Stata commands `etr` and `dietr` (Hebous and Mengistu, 2026).

*The views expressed here are those of the authors and not necessarily those of the IMF, its Executive Board, or IMF management.

[†]International Monetary Fund (Fiscal Affairs Department), Washington DC; CESifo, Munich; shebous@imf.org

[‡]International Monetary Fund (Fiscal Affairs Department), Washington DC; amengistu2@imf.org

0.1 Effective Tax Rates with Top-up (Pillar Two)

Domestic taxation follows DG sale timing (no immediate taxation of disposal gains). Pillar Two (GloBE) follows book (nominal) accounting, including book gains on disposal.

0.2 Minimum tax with two periods (with Qualified Tax Incentives (QTIs))

The rule agreed on January 6, 2026 provides that refundable tax credits—whether designated as QRTCs or NQRTCs—may be recharacterized as increases in covered taxes paid for GloBE purposes, subject to a quantitative cap. The cap is defined as the maximum of (i) 5.5 percent of payroll, (ii) 5.5 percent of depreciation, or (iii) 1 percent of qualifying expenditure (capital), if the taxpayer so elects.

Let

$$\overline{C} \equiv \max\{0.055 \cdot \text{payroll}, 0.055 \cdot \text{depreciation}, 0.01K_0\}, \quad C \equiv \min\{\text{Incentive}, \overline{C}\}.$$

Thus, only the portion C of the incentive can be treated as an increase in covered taxes for GloBE purposes.

Period-by-period implication. In each period, total tax liability is the sum of domestic tax and the GloBE top-up tax:

$$\text{Total tax} = \text{Domestic tax} + \text{GloBE top-up tax}. \quad (0.1)$$

(A) QRTC treated as covered taxes up to the cap

Under a QRTC, covered income is increased by the credit, but the amount treated as covered taxes is capped at $C = \min\{QRTC, \overline{C}\}$. The resulting total tax can be written as

$$\text{Total tax} = \tau\pi - QRTC + \left[\tau^{\min} - \frac{\tau\pi}{\pi + QRTC - C} \right] (\pi + QRTC - C - SBIE). \quad (0.2)$$

(B) NQRTC (treated as a reduction in taxes paid)

For comparison, let $NQRTC$ denote a non-qualified refundable credit (or a credit that is not treated as an increase in covered taxes). Under the same cap concept (i.e., only the portion $C = \min\{NQRTC, \overline{C}\}$ is treated as “qualified” for purposes of the adjustment), total tax can be expressed as

$$\text{Total tax} = \tau\pi - NQRTC + \left[\tau^{\min} - \frac{\tau\pi - NQRTC + C}{\pi} \right] (\pi - SBIE), \quad (0.3)$$

where $C = \min\{NQRTC, \overline{C}\}$ and $\overline{C} = \max\{0.055 \cdot \text{payroll}, 0.055 \cdot \text{depreciation}, 0.01K_0\}$.

Interpretation. If the incentive is *fully within the cap*—that is, $\text{Incentive} \leq \overline{C}$, so that $C = \text{Incentive}$ —the QTI treatment eliminates any distinction between QRTCs and NQRTCs for GloBE purposes: irrespective of their legal classification, both affect the minimum-tax computation only through the same capped adjustment C . When $\text{Incentive} > \overline{C}$, the portion of the incentive above the cap does not receive QTI treatment and therefore reintroduces a wedge between QRTCs and NQRTCs under the minimum tax.

Importantly, this does *not* imply that incentives above the cap are neutral for investment. Additional incentives beyond the cap may trigger a GloBE top-up, but because they further reduce domestic tax liabilities, they can still lower both the marginal and average effective tax

rates. As a result, incentives that exceed the cap may lead to even lower METRs and AETRs than incentives set exactly at the cap, despite increasing the GloBE top-up.

In the following, we highlight the METR and AETR for incentives under the cap. Let a fraction $\alpha \in [0, 1]$ of the investment be financed by debt. The nominal interest rate is i and the real required return is r , with $1 + i = (1 + r)(1 + \theta)$. Interest is deductible against domestic taxable income. Define the real (per-unit-of-capital) value of the domestic interest deduction as

$$\Psi \equiv \alpha \frac{i(1 - \tau\phi)}{1 + \theta}. \quad (0.4)$$

Environment and notation

$$1 + i = (1 + r)(1 + \theta), \quad (0.5)$$

$$A = \sum_{t \geq 0} \frac{\phi(1 - \phi)^t}{(1 + i)^t} = \phi \frac{1 + i}{i + \phi}, \quad (0.6)$$

$$\omega \equiv \frac{W_1}{P_1}, \quad (0.7)$$

$$\rho \equiv \frac{\tau^{\min} - \tau}{1 - \tau^{\min}}. \quad (0.8)$$

Qualified tax incentives (within the cap). We consider two incentive instruments, each assumed to be fully within the QTI cap and therefore excluded from covered income and covered taxes under Pillar Two. Let $qti^L \in [0, 1]$ and $qti^K \in [0, 1]$ denote utilization (or scaling) factors within the cap. The statutory cap parameter is common across instruments and fixed at $s_L = s_K \equiv 0.055$.

$$\mathcal{I}_1^L \equiv s_L qti^L W_1 L_1, \quad qti^L \in [0, 1], \quad s_L \equiv 0.055, \quad (0.9)$$

$$\mathcal{I}_1^K \equiv s_K qti^K \delta K_0, \quad qti^K \in [0, 1], \quad s_K \equiv 0.055. \quad (0.10)$$

The payroll-based incentive (Type L) distorts labor demand (through the labor FOC). The capital-based incentive (Type K) does not distort labor demand.

With CRS,

$$F(K_0, L_1) = K_0 f(\lambda), \quad \lambda \equiv \frac{L_1}{K_0}.$$

Define

$$H(\lambda) = f(\lambda) - \omega\lambda, \quad (0.11)$$

and the real operating return net of economic depreciation,

$$p(\lambda) = H(\lambda) - \delta. \quad (0.12)$$

GloBE excess profit

$$\begin{aligned} \mathcal{B}^G(K_0, L_1) &= [P_1 F(K_0, L_1) - W_1 L_1] + [(1 + \theta)(1 - \delta) - (1 - \phi)]K_0 \\ &\quad - (\phi + \gamma_K)K_0 - \alpha i(1 - \tau\phi) - \gamma_L W_1 L_1. \end{aligned} \quad (0.13)$$

$$S^G(\lambda) = P_1 [H(\lambda) - \gamma_L \omega \lambda] + [\theta - (1 + \theta)\delta - \alpha i(1 - \tau\phi) - \gamma_K]. \quad (0.14)$$

Value function

$$\begin{aligned}
V(K_0, L_1) &= -K_0 + \frac{1}{1+i} \left\{ (1-\tau)[P_1 F(K_0, L_1) - W_1 L_1] + (1-\delta)(1+\theta)K_0 \right\} + \tau A K_0 \\
&\quad - \frac{(1-\delta)(1+\theta)\tau A}{1+i} K_0 + \tau \alpha \frac{i(1-\tau\phi)}{1+i} + \frac{s_L q t i^L W_1 L_1}{1+i} + \frac{s_K q t i^K \delta K_0}{1+i} - \frac{\tau^u}{1+i} \max\{\mathcal{B}^G(K_0, L_1), 0\} \\
&= K_0 \mathcal{V}(\lambda).
\end{aligned} \tag{0.15}$$

$$\begin{aligned}
\mathcal{V}(\lambda) &= \frac{(1-\tau)H(\lambda)}{1+r} - (1-\tau A) \frac{r+\delta}{1+r} + \tau \alpha \frac{i(1-\tau\phi)}{1+i} \\
&\quad + \frac{s_L q t i^L \omega \lambda}{1+r} + \frac{s_K q t i^K \delta}{1+i} - \frac{\tau^u}{1+i} \max\{S^G(\lambda), 0\}.
\end{aligned} \tag{0.16}$$

Labor choice

Type L (payroll-based).

$$f'(\lambda_{NB}^L) = \left(1 - \frac{s_L q t i^L}{1-\tau}\right) \omega, \tag{0.17}$$

$$f'(\lambda_B^L) = \left(1 - \rho \gamma_L - \frac{s_L q t i^L}{1-\tau^{\min}}\right) \omega. \tag{0.18}$$

Type K (capital-based).

$$f'(\lambda_{NB}^K) = \omega, \tag{0.19}$$

$$f'(\lambda_B^K) = (1 - \rho \gamma_L) \omega. \tag{0.20}$$

Validity:

$$S^G(\lambda_{NB}) \leq 0, \quad S^G(\lambda_B) > 0.$$

Non-binding regime ($\tau^u \leq 0$) or excess profit is non-positive

Cost of capital.

$$\tilde{p}_{NB}^L = \frac{(r+\delta)(1-\tau A) - s_L q t i^L (\omega \lambda)_{NB}^L - \tau \alpha \frac{i(1-\tau\phi)}{1+\theta}}{1-\tau} - \delta, \tag{0.21}$$

$$\tilde{p}_{NB}^K = \frac{(r+\delta)(1-\tau A) - \frac{s_K q t i^K \delta}{1+\theta} - \tau \alpha \frac{i(1-\tau\phi)}{1+\theta}}{1-\tau} - \delta. \tag{0.22}$$

AETR.

$$AETR_{NB}^L(p) = \frac{\tau(p+\delta) - (r+\delta)\tau A - s_L q t i^L (\omega \lambda)_{NB}^L - \tau \alpha \frac{i(1-\tau\phi)}{1+\theta}}{p}, \tag{0.23}$$

$$AETR_{NB}^K(p) = \frac{\tau(p+\delta) - (r+\delta)\tau A - \frac{s_K q t i^K \delta}{1+\theta} - \tau \alpha \frac{i(1-\tau\phi)}{1+\theta}}{p}. \tag{0.24}$$

Binding regime ($\tau^u > 0$) or Excess Profit is Positive

Define

$$a \equiv \frac{\theta}{1+\theta} - \delta - \frac{\gamma_K}{1+\theta} - \alpha \frac{i(1-\tau\phi)}{1+\theta}.$$

Cost of capital.

$$\tilde{p}_B^L = \frac{(r + \delta)(1 - \tau A) + (\tau^{\min} - \tau)[a - \gamma_L(\omega\lambda)_B^L] - s_L q t i^L(\omega\lambda)_B^L - \tau\alpha \frac{i(1-\tau\phi)}{1+\theta}}{1 - \tau^{\min}} - \delta, \quad (0.25)$$

$$\tilde{p}_B^K = \frac{(r + \delta)(1 - \tau A) + (\tau^{\min} - \tau)[a - \gamma_L(\omega\lambda)_B^K] - \frac{s_K q t i^K \delta}{1+\theta} - \tau\alpha \frac{i(1-\tau\phi)}{1+\theta}}{1 - \tau^{\min}} - \delta. \quad (0.26)$$

AETR.

$$AETR_B^L(p) = \frac{\tau^{\min}(p + \delta) - (r + \delta)\tau A + (\tau^{\min} - \tau)[a - \gamma_L(\omega\lambda)_B^L] - s_L q t i^L(\omega\lambda)_B^L - \tau\alpha \frac{i(1-\tau\phi)}{1+\theta}}{p}, \quad (0.27)$$

$$AETR_B^K(p) = \frac{\tau^{\min}(p + \delta) - (r + \delta)\tau A + (\tau^{\min} - \tau)[a - \gamma_L(\omega\lambda)_B^K] - \frac{s_K q t i^K \delta}{1+\theta} - \tau\alpha \frac{i(1-\tau\phi)}{1+\theta}}{p}. \quad (0.28)$$

Cobb–Douglas specialization: explicit expression for $\omega\lambda$

Let

$$F(K, L) = \mathcal{A}K^\alpha L^{1-\alpha}, \quad 0 < \alpha < 1.$$

Under CRS,

$$F(K_0, L_1) = K_0 f(\lambda), \quad \lambda \equiv \frac{L_1}{K_0}, \quad f(\lambda) = \mathcal{A}\lambda^{1-\alpha}.$$

The marginal product of labor per unit of capital is

$$f'(\lambda) = (1 - \alpha)\mathcal{A}\lambda^{-\alpha}.$$

Recall that

$$H(\lambda) = f(\lambda) - \omega\lambda, \quad p(\lambda) = H(\lambda) - \delta,$$

so that

$$p(\lambda) + \delta = f(\lambda) - \omega\lambda. \quad (0.29)$$

Labor first-order condition. In all cases, the labor FOC can be written as

$$f'(\lambda) = \kappa\omega, \quad (0.30)$$

where the wedge κ depends on the regime and the instrument:

$$\begin{aligned} \kappa_{NB}^L &= 1 - \frac{s_L q t i^L}{1 - \tau}, & \kappa_B^L &= 1 - \rho\gamma_L - \frac{s_L q t i^L}{1 - \tau^{\min}}, \\ \kappa_{NB}^K &= 1, & \kappa_B^K &= 1 - \rho\gamma_L. \end{aligned}$$

Multiplying (0.30) by λ yields

$$\omega\lambda = \frac{\lambda f'(\lambda)}{\kappa}.$$

Under Cobb–Douglas,

$$\lambda f'(\lambda) = (1 - \alpha)\mathcal{A}\lambda^{1-\alpha} = (1 - \alpha)f(\lambda),$$

and therefore

$$\omega\lambda = \frac{1 - \alpha}{\kappa} f(\lambda). \quad (0.31)$$

Eliminating $f(\lambda)$. Substituting (0.31) into (0.29) gives

$$p(\lambda) + \delta = f(\lambda) - \frac{1-\alpha}{\kappa} f(\lambda) = f(\lambda) \frac{\kappa - (1-\alpha)}{\kappa}.$$

Solving for $f(\lambda)$,

$$f(\lambda) = \frac{\kappa}{\kappa - (1-\alpha)} (p + \delta).$$

Substituting back into (0.31) yields the closed form:

$$\boxed{\omega\lambda = \frac{1-\alpha}{\kappa - (1-\alpha)} (p + \delta).} \quad (0.32)$$

Define

$$m(\kappa) \equiv \frac{1-\alpha}{\kappa - (1-\alpha)}.$$

Explicit cases.

- **Non-binding, Type K :**

$$(\omega\lambda)_{\text{NB}}^K = \frac{1-\alpha}{\alpha} (p_{\text{NB}}^K + \delta).$$

- **Non-binding, Type L :**

$$(\omega\lambda)_{\text{NB}}^L = \frac{1-\alpha}{\alpha - \frac{s_L q t i^L}{1-\tau}} (p_{\text{NB}}^L + \delta).$$

- **Binding, Type K :**

$$(\omega\lambda)_{\text{B}}^K = \frac{1-\alpha}{\alpha - \rho\gamma_L} (p_{\text{B}}^K + \delta).$$

- **Binding, Type L :**

$$(\omega\lambda)_{\text{B}}^L = \frac{1-\alpha}{\alpha - \rho\gamma_L - \frac{s_L q t i^L}{1-\tau^{\min}}} (p_{\text{B}}^L + \delta).$$

Regularity. For $\omega\lambda$ to be well-defined and positive, the denominator must be positive, e.g.

$$\alpha > \frac{s_L q t i^L}{1-\tau} \quad \text{and} \quad \alpha > \rho\gamma_L \quad (\text{and analogously in the binding case}).$$

Closed-form results under Cobb–Douglas

Cost of Capital

Non-binding.

$$\tilde{p}_{\text{NB}}^L = \frac{(r + \delta)(1 - \tau A) - \tau\alpha \frac{i(1-\tau\phi)}{1+\theta}}{(1 - \tau) + s_L q t i^L m_{\text{NB}}^L} - \delta, \quad (0.33)$$

$$\tilde{p}_{\text{NB}}^K = \frac{(r + \delta)(1 - \tau A) - \frac{s_K q t i^K}{1+\theta} \delta - \tau\alpha \frac{i(1-\tau\phi)}{1+\theta}}{1 - \tau} - \delta. \quad (0.34)$$

Binding.

$$\tilde{p}_B^L = \frac{(r + \delta)(1 - \tau A) + (\tau^{\min} - \tau)a - \tau\alpha \frac{i(1-\tau\phi)}{1+\theta}}{(1 - \tau^{\min}) + m_B^L[(\tau^{\min} - \tau)\gamma_L + s_L qti^L]} - \delta, \quad (0.35)$$

$$\tilde{p}_B^K = \frac{(r + \delta)(1 - \tau A) + (\tau^{\min} - \tau)a - \frac{s_K qti^K \delta}{1+\theta} - \tau\alpha \frac{i(1-\tau\phi)}{1+\theta}}{(1 - \tau^{\min}) + m_B^K(\tau^{\min} - \tau)\gamma_L} - \delta. \quad (0.36)$$

AETR

Non-binding (NB).

Type L (payroll-based).

$$AETR_{NB}^L(p) = \frac{\left(\tau - s_L qti^L m_{NB}^L\right)(p + \delta) - (r + \delta)\tau A - \tau\alpha \frac{i(1-\tau\phi)}{1+\theta}}{p} \quad (0.37)$$

Type K (capital-based).

$$AETR_{NB}^K(p) = \frac{\tau(p + \delta) - (r + \delta)\tau A - \frac{s_K qti^K \delta}{1+\theta} - \tau\alpha \frac{i(1-\tau\phi)}{1+\theta}}{p} \quad (0.38)$$

Binding (B).

Type L (payroll-based).

$$AETR_B^L(p) = \frac{\left(\tau^{\min} - m_B^L[(\tau^{\min} - \tau)\gamma_L + s_L qti^L]\right)(p + \delta) - (r + \delta)\tau A + (\tau^{\min} - \tau)a - \tau\alpha \frac{i(1-\tau\phi)}{1+\theta}}{p} \quad (0.39)$$

Type K (capital-based).

$$AETR_B^K(p) = \frac{\left(\tau^{\min} - (\tau^{\min} - \tau)\gamma_L m_B^K\right)(p + \delta) - (r + \delta)\tau A + (\tau^{\min} - \tau)a - \frac{s_K qti^K \delta}{1+\theta} - \tau\alpha \frac{i(1-\tau\phi)}{1+\theta}}{p} \quad (0.40)$$

Elasticities

In the non-binding regime with a payroll-based incentive (Type L), we have

$$\tilde{p}_{NB}^L + \delta = \frac{(r + \delta)(1 - \tau A)}{(1 - \tau) + s_L qti^L m_{NB}^L}. \quad (0.41)$$

Define the usual user cost of capital

$$R \equiv \frac{(r + \delta)(1 - \tau A)}{1 - \tau}, \quad \Rightarrow \quad (r + \delta)(1 - \tau A) = R(1 - \tau). \quad (0.42)$$

Let the effective payroll-subsidy intensity be

$$S \equiv s_L qti^L. \quad (0.43)$$

Under Cobb–Douglas, the non-binding labor wedge is

$$\kappa_{\text{NB}}^L = 1 - \frac{S}{1 - \tau},$$

so that

$$m_{\text{NB}}^L = \frac{1 - \alpha}{\kappa_{\text{NB}}^L - (1 - \alpha)} = \frac{1 - \alpha}{\alpha - \frac{S}{1 - \tau}}. \quad (0.44)$$

Substituting (0.42) into (0.41) yields

$$p + \delta \equiv \tilde{p}_{\text{NB}}^L + \delta = \frac{R(1 - \tau)}{(1 - \tau) + S m_{\text{NB}}^L}. \quad (0.45)$$

Using (0.44) to eliminate m_{NB}^L and simplifying gives the closed form

$$p + \delta = R \frac{\alpha(1 - \tau) - S}{\alpha((1 - \tau) - S)}. \quad (0.46)$$

Taking logs,

$$\log(p + \delta) = \log R + \log(\alpha(1 - \tau) - S) - \log \alpha - \log((1 - \tau) - S). \quad (0.47)$$

Elasticity with respect to R . Holding S fixed, differentiate (0.47) with respect to $\log R$:

$$\frac{d \log(p + \delta)}{d \log R} = 1. \quad (0.48)$$

Equivalently, in differential form,

$$d \log(p + \delta) = d \log R = \frac{dR}{R}. \quad (0.49)$$

Semi-elasticity with respect to the payroll subsidy S (or s_L when $qti^L = 1$). Holding R fixed, differentiate (0.47) with respect to S :

$$\begin{aligned} \frac{d \log(p + \delta)}{dS} &= \frac{d}{dS} \log(\alpha(1 - \tau) - S) - \frac{d}{dS} \log((1 - \tau) - S) \\ &= -\frac{1}{\alpha(1 - \tau) - S} + \frac{1}{(1 - \tau) - S}. \end{aligned} \quad (0.50)$$

Thus,

$$\frac{d \log(p + \delta)}{dS} = \frac{1}{(1 - \tau) - S} - \frac{1}{\alpha(1 - \tau) - S}. \quad (0.51)$$

If $qti^L = 1$, then $S = s_L$ and (0.51) is also the semi-elasticity with respect to s_L :

$$\frac{d \log(p + \delta)}{ds_L} = \frac{1}{(1 - \tau) - s_L} - \frac{1}{\alpha(1 - \tau) - s_L}.$$

Regularity. For the expressions above to be well-defined (and for $m_{\text{NB}}^L > 0$), it is sufficient that

$$S < \alpha(1 - \tau), \quad \text{and} \quad S < (1 - \tau).$$

Starting from the semi-elasticity with respect to the payroll subsidy SL ,

$$\frac{d \log(p + \delta)}{dSL} = \frac{1}{(1 - \tau) - SL} - \frac{1}{\alpha(1 - \tau) - SL}, \quad (0.52)$$

combine the two terms into a single fraction:

$$\begin{aligned} \frac{d \log(p + \delta)}{dSL} &= \frac{(\alpha(1 - \tau) - SL) - ((1 - \tau) - SL)}{((1 - \tau) - SL)(\alpha(1 - \tau) - SL)} \\ &= \frac{(\alpha - 1)(1 - \tau)}{((1 - \tau) - SL)(\alpha(1 - \tau) - SL)} \\ &= -\frac{(1 - \alpha)(1 - \tau)}{((1 - \tau) - SL)(\alpha(1 - \tau) - SL)}. \end{aligned} \quad (0.53)$$

Evaluating at $SL = 0$ yields

$$\left. \frac{d \log(p + \delta)}{dSL} \right|_{SL=0} = -\frac{1 - \alpha}{\alpha(1 - \tau)}. \quad (0.54)$$

Therefore, the equivalent change in SL compared to R is given by:

$$\frac{\Delta R}{R} = -\frac{1 - \alpha}{\alpha(1 - \tau)} \Delta SL. \quad (0.55)$$

1 Annex: Demonstration of the Generosity of the Payroll Incentive

To illustrate the generosity of payroll-based tax incentives, this annex abstracts from the global minimum tax and considers a purely domestic investment subject only to the corporate income tax. We derive the cost of capital and the marginal effective tax rate (METR) in the presence of a payroll-based credit.

1.1 Payroll-Based Tax Credits, User Cost, and the METR

Consider a firm that chooses capital K and labor L to maximize after-tax value

$$V = (1 - \tau) \left[F(K, L) - wL - \frac{(r + \delta)(1 - \tau A_{\text{npv}})}{1 - \tau} K \right] + S w L, \quad (1.1)$$

where τ is the statutory corporate income tax rate, r the real required rate of return, δ the economic depreciation rate, A_{npv} the present value of depreciation allowances per unit of investment, S a payroll-based tax credit expressed as a fraction of the wage bill, and w the real wage.

The production technology is Cobb–Douglas:

$$F(K, L) = K^\alpha L^\beta, \quad \alpha > 0, \beta > 0, \alpha + \beta < 1. \quad (1.2)$$

Labor Choice

The first-order condition with respect to labor is

$$\frac{\partial V}{\partial L} = (1 - \tau) [F_L(K, L) - w] + S w = 0, \quad (1.3)$$

which, using $F_L = \beta K^\alpha L^{\beta-1}$, implies

$$\beta K^\alpha L^{\beta-1} = w \left(1 - \frac{S}{1 - \tau} \right). \quad (1.4)$$

Solving for labor as a function of capital,

$$L(K) = \left(\frac{\beta}{w \left(1 - \frac{S}{1 - \tau} \right)} \right)^{\frac{1}{1-\beta}} K^{\frac{\alpha}{1-\beta}}. \quad (1.5)$$

The payroll subsidy therefore acts as a wedge that lowers the effective wage faced by the firm.

Reduced-Form Operating Surplus

Substituting (1.5) into production and wage payments yields

$$F(K, L(K)) - wL(K) = (1 - \beta) \beta^{\frac{\beta}{1-\beta}} w^{-\frac{\beta}{1-\beta}} \left(1 - \frac{S}{1 - \tau} \right)^{-\frac{\beta}{1-\beta}} K^{\frac{\alpha}{1-\beta}}. \quad (1.6)$$

All dependence on the payroll subsidy enters multiplicatively through $\left(1 - \frac{S}{1 - \tau} \right)^{-\frac{\beta}{1-\beta}}$.

Capital Choice

Substituting (1.6) into (1.1) gives

$$V = (1 - \tau) \Omega \left(1 - \frac{S}{1 - \tau}\right)^{-\frac{\beta}{1-\beta}} K^{\frac{\alpha}{1-\beta}} - (r + \delta)(1 - \tau A_{\text{npv}})K, \quad (1.7)$$

where

$$\Omega = (1 - \beta) \beta^{\frac{\beta}{1-\beta}} w^{-\frac{\beta}{1-\beta}}$$

is constant with respect to K , S , and τ .

The first-order condition with respect to capital implies

$$(1 - \tau) \Omega \left(1 - \frac{S}{1 - \tau}\right)^{-\frac{\beta}{1-\beta}} \frac{\alpha}{1 - \beta} K^{\frac{\alpha+\beta-1}{1-\beta}} = (r + \delta)(1 - \tau A_{\text{npv}}). \quad (1.8)$$

Solving for optimal capital,

$$K^* = \left[\frac{(1 - \tau) \Omega \frac{\alpha}{1-\beta}}{(r + \delta)(1 - \tau A_{\text{npv}})} \right]^{\frac{1-\beta}{1-\alpha-\beta}} \left(1 - \frac{S}{1 - \tau}\right)^{-\frac{\beta}{1-\alpha-\beta}}. \quad (1.9)$$

Log Representation

Define the standard user cost of capital

$$R = \frac{(r + \delta)(1 - \tau A_{\text{npv}})}{1 - \tau}.$$

Taking logs of (1.9),

$$\log K^* = \text{const} - \frac{1 - \beta}{1 - \alpha - \beta} \log R - \frac{\beta}{1 - \alpha - \beta} \log \left(1 - \frac{S}{1 - \tau}\right). \quad (1.10)$$

Thus, the payroll subsidy enters *exactly like a reduction in the user cost of capital*.

Elasticities

The semi-elasticity of capital with respect to the payroll subsidy is

$$\frac{\partial \log K^*}{\partial S} = \frac{\beta}{1 - \alpha - \beta} \frac{1}{(1 - \tau) - S}, \quad (1.11)$$

while the semi-elasticity with respect to the user cost is

$$\frac{\partial \log K^*}{\partial R} = -\frac{1 - \beta}{1 - \alpha - \beta} \frac{1}{R}. \quad (1.12)$$

1.2 Comparing the Effect of a Change in the User Cost with a Payroll Subsidy

Equating the effects of a change in S and a proportional change in R on $\log K^*$ implies

$$\frac{\Delta R}{R} = -\frac{\beta}{1 - \beta} \left[\frac{\Delta S}{(1 - \tau) - S} \right] \quad (1.13)$$

Equation (1.13) shows that a payroll-based subsidy is equivalent to a proportional reduction in the user cost of capital, with amplification factor $\beta/(1 - \beta)$.

Note that in the constant returns to scale DG version, $\beta = 1 - \alpha$ and the $1 - \beta = \alpha$. This shows that the formulas we derived in equation is equivalent to (1.13)

Numerical Illustration

Assume

$$r = 0.05, \quad \delta = 0.25, \quad \pi = 0.05, \quad \tau = 0.25, \quad S = 0.055.$$

The nominal interest rate is

$$i = (1 + r)(1 + \pi) - 1 = (1.05)(1.05) - 1 = 0.1025.$$

The present value of depreciation allowances (declining-balance) is

$$A_{\text{npv}} = \delta \frac{1 + i}{i + \delta} = 0.25 \cdot \frac{1.1025}{0.3525} \approx 0.7819.$$

The corresponding user cost of capital is

$$R = \frac{(r + \delta)(1 - \tau A_{\text{npv}})}{1 - \tau} = \frac{0.30(1 - 0.25 \times 0.7819)}{0.75} \approx 0.3218.$$

To gauge the generosity of the payroll-based incentive, take $\beta = 0.6$ and consider a discrete increase in the payroll credit from $S = 0$ to $S = 0.055$. Using (1.13), the equivalent proportional change in the user cost is

$$\frac{\Delta R}{R} \approx -\frac{0.6}{0.4} \cdot \frac{0.055}{0.75} = -1.5 \cdot 0.073\bar{3} \approx -0.110,$$

i.e. an effective *11.0% reduction* in the user cost of capital.

Implications for the METR

The marginal effective tax rate (METR) is defined as

$$\text{METR} = \frac{R - (r + \delta)}{r}. \tag{1.14}$$

Since r is fixed, changes in the user cost map one-for-one into changes in the METR:

$$\Delta \text{METR} = \frac{\Delta R}{r} = \frac{R}{r} \cdot \frac{\Delta R}{R}. \tag{1.15}$$

The baseline METR is therefore

$$\text{METR}_0 = \frac{0.3218 - 0.30}{0.05} \approx 0.436,$$

or 43.6 percent. Applying the payroll credit reduces the user cost to

$$R_1 = R \left(1 + \frac{\Delta R}{R} \right) \approx 0.3218 \times (1 - 0.110) \approx 0.286.$$

The implied METR becomes

$$\text{METR}_1 = \frac{0.286 - 0.30}{0.05} \approx -0.28.$$

Hence, a payroll credit equal to 5.5 percent of the wage bill can reduce the METR by roughly 70 percentage points in this calibration, turning it negative.

How Results Vary with τ and β

The equivalence (1.13) implies stronger effects when (i) the labor exponent β is larger (since $\beta/(1-\beta)$ rises sharply), and (ii) the statutory rate τ is higher (since $S/(1-\tau)$ increases as $1-\tau$ shrinks). Using the same (r, δ, π, S) and the same A_{npv} , the table below reports the implied $\Delta R/R$ and the resulting METR.

τ	β	R_0	$\Delta R/R$	R_1	METR ₀	METR ₁
0.25	0.50	0.3218	-0.0733	0.2982	0.4362	-0.0358
0.10	0.50	0.3073	-0.0611	0.2885	0.1454	-0.2302
0.10	0.70	0.3073	-0.1426	0.2635	0.1454	-0.7309
0.05	0.50	0.3034	-0.0579	0.2859	0.0689	-0.2825
0.05	0.70	0.3034	-0.1351	0.2625	0.0689	-0.7510

In particular, the added case $(\tau, \beta) = (0.25, 0.5)$ shows that even with a more moderate labor exponent, a 5.5% payroll credit can still be equivalent to about a 7.3% reduction in the user cost and can push the METR slightly below zero in this calibration.